A generalized linear Hubble law for an inhomogeneous barotropic Universe

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Abstract

In this work, I present a generalized linear Hubble law for a barotropic spherically symmetric inhomogeneous spacetime, which is in principle compatible with the acceleration of the cosmic expansion obtained as a result of high redshift Supernovae data. The new Hubble function, defined by this law, has two additional terms besides an expansion one, similar to the usual volume expansion one of the FLRW models, but now due to an angular expansion. The first additional term is dipolar and is a consequence of the existence of a kinematic acceleration of the observer, generated by a negative gradient of pressure or of mass-energy density. The second one is quadrupolar and due to the shear. Both additional terms are anisotropic for off-centre observers, because of to their dependence on a telescopic angle of observation. This generalized linear Hubble law could explain, in a cosmological setting, the observed large scale flow of matter, without to have recourse to peculiar velocity-type newtonian models. It is pointed out also, that the matter dipole direction should coincide with the CBR dipole one.

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1 Introduction

Last year [1], I have considered a specific inhomogeneous cosmological model, which could explain, in an alternative way, by the presence of a kinematic acceleration generated by a negative gradient of pressure (or mass-energy), the present negative deceleration parameter, which appears to be a result of high redshift Supernovae data. These data were obtained by two different groups [2,3] and, if the SNe Ia are standard candles without evolution effects (although a recent work [4] suggests indicate that the SNe Ia are not standard candles), they show the existence of a cosmic acceleration, via the K-corrected redshift-luminosity distance relation. In the usual way, these data are explained, in the background of standard Friedmann models, by the presence of a positive cosmological constant or vacuum energy or quintessence.

This specific inhomogeneous spacetime, which I will also consider in this work, is a barotropic (B), spherically (S) symmetric (hereafter BS). The comoving matter congruence, in the vorticity-free BS inhomogeneous model, has expansion, kinematic acceleration and shear, at difference with the standard FLRW models, where it has expansion only.

At first sight, it seems that spherically symmetric inhomogeneous models are non generic among the totality of inhomogeneous ones, but note however that they can appear as a natural consequence of inflation (see [5]). Note also that the measured almost isotropy of the cosmic background radiation (CBR) temperature is in principle compatible with large shear [6] or nonzero kinematic acceleration [7], although these possibilities have yet to be tested against the full range of cosmological observations, including CBR polarization.

As the Cosmological Principle (CP) is not verified in the BS model and, as usually the expansion linear Hubble's law is derived from the CP, one can ask how this law, which characterizes the matter flow, is generalized by the additional presence of acceleration and shear in the BS model. Fortunately, the general linear Hubble law, valid for any inhomogeneous Universe, is at our disposal. For instance, it was presented in

[8,9] and its expression, to linear order in the angular diameter distance, is:

$$z = \left(\frac{\theta}{3} - \dot{u}_a \, o^a + \sigma_{ab} \, o^a o^b \right) D \qquad (1)$$

where z is the measured redshift, D is the angular diameter distance to the emitter (however, note that, at linear order, the Hubble law can be expressed in terms of any cosmological distance, see [16]), θ the expansion, $\dot{u}_a$ is the 4-kinematic acceleration of a comoving observer, σ_{ab} is the shear tensor and o^a, is an unit vector defined in observer's rest space, which points in the observed direction of a light ray and, hence, is opposite to the motion sense of the ray's photons.

This formula directly shows the isotropic, direccional and fully anisotropic contributions to the redshift from the expansion, acceleration and shear, respectively. All the terms between parenthesis can be interpreted as a generalized Hubble function. The first term, due to the cosmological expansion, gives the usual expansion linear Hubble law of the FLRW models, which is sometimes reinterpreted at small distances as a radial Doppler effect. Although, we are opposed to a newtonian kinematic reinterpretation at large distances of the cosmological expansion redshift, however, it is clear that the expansion redshift can be modified by nearly constant Doppler and gravitational contributions, at both ends (and also in the light travel) of the emission-observation process.

2   The BS model

In order to specialize the general formula (1) to the BS model, we need to summarize its main characteristics. We assume from the beginning that the cosmological matter fluid is perfect and a mixture (radiation, baryonic matter, neutrinos, etc.) and hence is characterized by a non-dust perfect fluid congruence of unit 4-velocity u^a. In the BS model, the isotropy group is 1-dim, whereas the isometry group is a 3-dim G_3, which is acting multiply transitively on spacelike 2-dim surfaces orthogonal to a preferred direction (hereafter e_1). These 2-dim surfaces orbits, orthogonal both to the 4-velocity of the congruence and to e_1, have constant Gaussian curvature but, of course, they can be spheres, hyperboloids or planes. We consider, in the BS model, the case of spherical

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symmetry.

Also, in the BS model the vorticity is zero. This implies both the existence of a global cosmic time and of 3-dim global spacelike hypersurfaces orthogonal to the fluid congruence. In the BS model, the preferred spacelike direction e_1 is not only geodesic and shear free in the local rest spaces orthogonal to the 4-velocity u^a, moreover e_1 is geodesic along the matter flow lines.

Thus, the metric in comoving coordinates has the expression

$$ds^2 = -N^2(r,t)\,dt^2 + B^2(r,t)\,dr^2 + R^2(r,t)\,d\Omega^2. \tag{2}$$

where $d\Omega^2$ is the spherical line element and the coefficient $N(r,t)$ is a lapse function which relates global cosmic and local proper times. So, we have adopted in the 1+3 threading approach the Lagrangian formalism, with the comoving identification, but the matter observers are Eulerian (due to the null vorticity) in the ADM 3+1 slicing formalism.

In the BS spacetime exists a preferred central worldline, i.e. the spacelike hypersurfaces have a centre at $r = 0$, where the isotropy group is 3-dim, and a preferred radial direction e_1 at each point, associated with the direction of the only non-null component, A, of the kinematic acceleration of the matter fluid elements. This kinematic acceleration satisfies the spatially contracted equations of motion

$$p' + (\mu + p)\,A = 0, \tag{3}$$

where a prime denotes the derivative along the preferred radial direction with unit vector field e_1. As we assume a barotropic equation of state, $p(r,t) = p(\mu(r,t))$, with $\mu(r,t)$ the mass-energy density, the last equation may also be written as:

$$c_s^2\,\mu' + (\mu + p)\,A = 0, \tag{4}$$

where c_s is the sound velocity. We see from the last formulas that the kinematic acceleration, which opposes to the gravitational attraction towards the centre, is generated by a negative gradient of pressure or, equivalently, by a negative gradient of mass-energy density. Note (see [1]), that the presence in the metric of the coefficient $N(r,t)$, gives rise to a non null kinematic acceleration, defined in the next section, and also to

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a new term in the expression of the deceleration parameter q, which was called in [1], inhomogeneity parameter, and was defined there as

$$\mathcal{I} = \frac{\alpha(t)}{(S\,H)^2},$$ (5)

where $\alpha(t)$ is the coefficient of the second order term in the radial expansion of the metric coefficient $N(r,t)$, and where S and H, are the scale expansion factor and the usual expansion Hubble function of FLRW models, respectively.

Depending on the value of this additional inhomogeneity term, the deceleration parameter q can be positive or negative at present cosmic time. In the last case, one has an alternative explanation for the Supernovae data, in the realm of the BS model, as it has been commented before.

3   Generalized Hubble law for the BS model

Now, we specialize the general Hubble law (1), for the BS model considered. Besides the null vorticity, the remaining kinematical quantities have the following characteristics: the scalar expansion $\theta(r,t)$ is a function of a radial comoving coordinate and cosmic time, the 4-acceleration of the non-geodesic comoving observers is $\dot{u}_a = (0, A(r,t), 0, 0)$, and the shear mixed tensor has the diagonal coordinate expression $\sigma_a^b = \mathrm{diag}(0, \frac{2}{\sqrt{3}}\sigma(r,t), -\frac{1}{\sqrt{3}}\sigma(r,t), -\frac{1}{\sqrt{3}}\sigma(r,t))$, where $\sigma^2 = \frac{1}{2}\sigma_a^b\sigma_b^a$.

The temporal component of both acceleration and shear is zero by the choice of the comoving frame. On the other hand, the 4-acceleration has only one non-zero component in the sense of the preferred spatial direction e_1 away from the centre, and the shear mixed tensor the above diagonal form. Finally, due to spherical symmetry, their components are only functions of r and cosmic time t. The expressions of the scalars θ, A and σ, are

$$\theta(r,t) = \frac{\dot{B}}{B} + 2\frac{\dot{R}}{R},$$ (6)

$$A(r,t) = \frac{N'}{N}$$ (7)

and

$$\sigma(r,t) = \frac{1}{\sqrt{3}}\left(\frac{\dot{B}}{B} - \frac{\dot{R}}{R}\right),$$ (8)

where the point means the proper time derivative, $\frac{1}{N}\frac{\partial}{\partial t}$. So, the scalar shear σ is, except by a constant factor, the difference between the radial and azimuthal expansion rates.

Now, performing a similar computation to that realized in [10] for a dust subcase (Tolman-Bondi) of the BS model, the final expression that adopts the linear Hubble's law in the BS model, for off-centre observers P_0, is:

$$z = \left(\frac{\dot{R}}{R} - A\cos\Psi + \sqrt{3}\,\sigma\cos^2\Psi \right)_0 D \tag{9}$$

By spherical symmetry, just the telescopic angle Ψ is needed to describe off-centre observations in the rest space of the observer. Ψ is the angle between the direction of observation of a light ray, o^a, and the vector e_1, pointing radially outward from the centre in the spacelike hypersurfaces of the BS model.

The generalized Hubble function has three terms which contribute to the cosmological redshift. The first is similar to the usual one of FLRW models due to the volume expansion, but in this model is due to the azimuthal expansion and depends not only on cosmic time but also on a radial comoving distance or position of the observer with respect to the centre. The azimuthal expansion comes from the expansion of the geometrical S^2 sphere centered at the observer $(r = r_0)$ in the spacelike hipersurface $(t = t_0)$. Thus, the azimuthal expansion implies a variation in cosmic time of the angle Ψ, under which a specific emitter is observed in the observational celestial sphere of the rest space of the observer. The second term in the generalized Hubble function is a dipolar one, due to the acceleration (which does not appear in the Tolman-Bondi subcase), and a third quadrupolar one, due to the shear.

The inhomogeneous BS model has a preferred central worldline (a spatial centre in the spacelike hypersurfaces) in which the mass-energy density and the pressure are global maxima at any cosmic time, and where the acceleration and the shear are zero. Hence, at the centre, the matter congruence and CBR are exactly isotropic, but for off-centre observers as us, additional terms appear due to the radial inhomogeneity of this spacetime.

The consequences of the generalized linear Hubble law are striking. For instance,

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when we are observing in the same sense that the acceleration away from the centre, that is $\Psi = 0$, then the acceleration term gives a maximum violetshift contribution, $-AD$, due to the fact that we are expelled out from the centre by the kinematic acceleration. Of course, observing in the opposite sense to the acceleration, i.e., towards the centre, it gives an maximum dipole redshift, $+AD$. In both cases, if the emitters are at the same distance, the additional expansion and shear quadrupole terms have the same positive value. Hence, using the new Hubble law (9), the difference of redshifts of these kind of observations, is a pure dipole violetshift, which is only due to the kinematic acceleration A. Therefore, we consider this specific direction away from the centre, as the global direction of the matter dipole.

Moreover, as it have been stated before, the dipole term can also be modified by nearly constant dipole redshifts originated by Doppler and gravitational contributions in the process of emission-observation of light. By using the BS model, these additional redshifts must be calculated as deviations from the generalized Hubble law (9) valid for comoving non-geodesic observers. However, as it has been stated above, for isolating the pure dipole part, it is necessary to make the difference between redshift observations of emitters situated at the same distance, but in opposite senses from the observer.

Additional dipole voiletshifts are normally interpreted, by using Newton's theory, as deviations from an isotropic linear Hubble expansion law due to peculiar velocity fields or a bulk flow towards a local "Great Attractor", (see [11]).

However, in the model considered, the centre of the spacelike 3-dim hypersurfaces is the "global repulsor" and is the centre of the rest-frame of matter and also of the rest-frame of the CBR. Hence the matter dipole direction given by the generalized Hubble law must coincide with the CBR dipole direction. With respect to the CBR dipole, we refer to a recent work [12], where all the anisotropies including non-linear effects have been calculated, and where it was estimated, how the kinematic acceleration contributes to the dipole temperature anisotropy and the shear to the quadrupole one. I leave for future work the obtention of the CBR dipole and quadrupole temperatures in the BS model and the influence of the new form of the Hubble function on its deceleration parameter.

4   Final comments and conclusions

Other works which have tried to explain the matter or the CBR dipole as a cosmological effect (see for instance [10,13,14]), have considered a matter congruence of geodesic observers, because all use the dust Tolman-Bondi spacetime (except the recent work [15] on a special Stephani spacetime). Instead, in the BS model, the gradient of pressure is not zero, and hence, the congruence of matter observers is non-geodesic, except at the centre, in which it is supposed to be the origin of the matter flow. Note that, even in the late Universe, when the pressure is small (if the mean velocity of Galaxies is around 300 Km/sec or $10^{-3} c$, then the pressure is around 10^{-6} of the mass-energy density), however, the gradient of pressure could not be necessarily small.

The main difference between the cosmological matter dipole of the new law (9) and the usual dipole, manifested as a deviation of the isotropic Hubble law of Friedmann models, is that the latter is interpreted as a constant Doppler effect, i.e. independent of the emitter's distance. This Doppler dipole arises in the standard model from peculiar velocities, and it can be eliminated going to the "correct" zero peculiar velocity frame. Whereas, the cosmological acceleration dipole, measured off the centre in the fundamental comoving frame of non-geodesic matter observers, is emitter's distance dependent and cannot be eliminated for off-centre observers. The same happens for the quadrupole shear term.

However, we do not claim that all the observed matter dipole is cosmological. In our model, a peculiar velocity dipole induced by a local inhomogeneity, must be calculated as a local perturbation of the inhomogeneous background spacetime, i.e. as a local deviation of the new Hubble law (9).

Finally note, that the new Hubble function of this model is not only cosmic time dependent, as in FLRW models, but observer's position and angular dependent too. Therefore, this may account for the difference between its inferred values from observations performed with different telescopic angles.

Acknowledgements

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